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# A NOVEL ALGORITHM FOR THE SIGNAL INTERPOLATION OF THE DISPLACEMENT MEASUREMENT BASED ON A FABRY-PEROT INTERFEROMETER

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## ABSTRACT

Current commercial interferometers reveal excellent measurement performances, because of its major advantage which enables the displacement measurement with the characterization of the high resolution under the large measuring range. Fabry-Perot interferometer is a compact interferometer with the structure of common optical path. In comparison with the ordinary commercial laser interferometers having non-common optical path, Fabry-Perot interferometer is more insensitive to environmental disturbances. But the disadvantages of Fabry-Perot interferometer are the limited measuring range and the considerable visibility decadence of the interference pattern.

To enlarge the measuring range, the folded Fabry-Perot interferometer in which a corner cube reflector serves as the measurement mirror has been proposed in the previous research. However, either the conventional Fabry-Perot interferometer or the folded Fabry-Perot interferometer still have the problem of the considerable visibility decadence of the interference pattern. When the length of optical cavity is changed, the overlapped state of the laser beams will be varied strongly. Hence, it is indispensable to offer a proper signal interpolation algorithm for various signal distributions during the displacement measurement in the whole measuring range.

An advanced signal interpolation algorithm for the above-mentioned Fabry-Perot interferometer has been proposed in this investigation. The novel algorithm is able to solve the problem of the displacement measurement due to the considerable visibility decadence of the interference pattern. With this algorithm, a high precision displacement measurement in the large measuring range can be realized by the folded Fabry-Perot interferometer.

**Index Terms** - Folded Fabry-Perot interferometer, Signal interpolation algorithm, Displacement measurement

## 1. INTRODUCTION

Fabry-Perot interferometer (Fig. 1) is a kind of interferometer with the common optical path. In some shown researches, the interferometer with common optical path structure can be more independent of environment disturbances and vibration in some situation [1, 2].

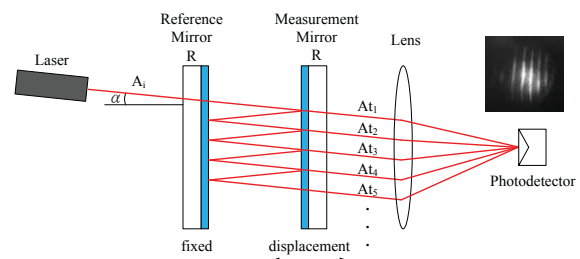


Figure 1 - Typical Fabry-Perot Interferometer

However, the signal models of the Fabry-Perot interferometer are not always identical. The signal will change into different finesse with the different lengths of cavity, incident angles of the light source and tilt angles of the two interference mirrors. Because of the limitation of the mechanism, a uniform signal model is not a feasible solution. For the precision of the displacement measurement by the Fabry-Perot interferometer, a variable signal interpolation algorithm must be arranged.

## 2. THEORY AND PRINCIPLE

The theory of Fabry-Perot interferometer has been investigated in many researches since 1897. Most signal models are based on the situation of short traveling range or the ideal condition. In this study, a novel signal processing for large measuring range by Fabry-Perot interferometer will be described.

### 2.1. Original Fabry-Perot interferometer

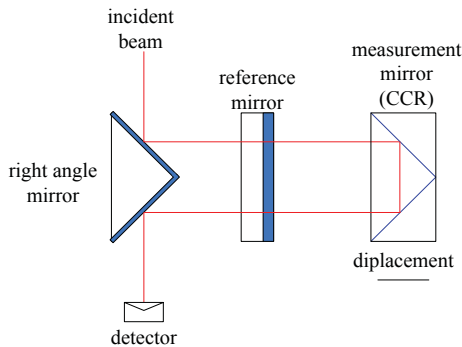
The original theory of Fabry-Perot interferometer is proposed by Charles Fabry and Alfred Perot in 1897. The equation of the luminous intensity of the transmission light ( $I_1$ ) is derived as Eq. 1 ( $I_0$  is the luminous intensity of incident laser source,  $R$  and  $T$  is the reflectance and transmittance of the reference and measurement mirror,  $d$  is the distance of the cavity,  $\lambda$  is the wavelength of the light source). The typical structure of Fabry-Perot interferometer cavity is based on two approximately parallel planar mirrors (Fig. 1). The parallelism of the two mirrors is a critical parameter for the measuring range. For this reason, the typical Fabry-Perot interferometer is difficult to perform displacement measurement in the large range.

$$I_1 = \frac{I_0 \times T^2}{1 + R^2 - 2 \times R \times \cos\left(\frac{4\pi d}{\lambda}\right)} \quad (1)$$

### 2.2. Folded Fabry-Perot interferometer

In 1961, P. Rabinowitz et al. [3] proposed to use the corner cube reflector as the measurement mirror of the Fabry-Perot interferometer. By this way, the deterioration of the interferometer signal which results from the tilt angle of the measurement mirror can be avoided. Thus, the measuring range of Fabry-Perot interferometer can be enhanced to hundred millimeters [4]. The equation of the luminous intensity of the transmission light ( $I_2$ ) is derived as Eq. 2.

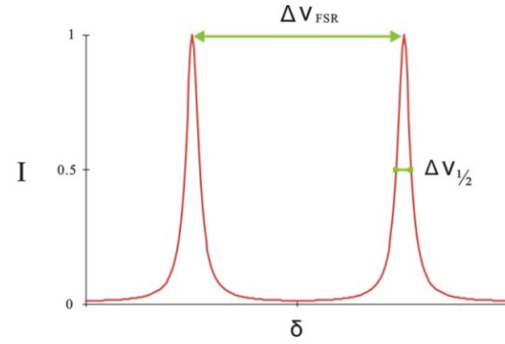
$$I_2 = \frac{I_0 \times T^2}{1 + R^2 - 2 \times R \times \cos\left(\frac{8\pi d}{\lambda}\right)} \quad (2)$$



**Figure 2 - Folded Fabry-Perot Interferometer**

### 2.3. Definition of finesse

The intensity equations which are mentioned in paragraph 2.1 and 2.2 are derived under the ideal condition. The incident angle is approximately zero and the parallelism of the optical cavity is almost perfect. In the real situation, there are a lot of parameters which include incident angle, parallelism of the optical cavity, beam size of the light source and distribution type of the light source etc. should be considered in the deduction. Hence, many previous researches dealt with the parameters of the Fabry-Perot interferometer [5]. And all parameters can be inducted into one parameter which is called finesse ( $\mathcal{F}$ ). Finesse is one of the most important parameters in the optical cavity. The definition of finesse is shown in Fig. 3 and Eq. 3. The finesse can represent the fringe pattern of the Fabry-Perot interferometer.



**Figure 3 - Definition of finesse**

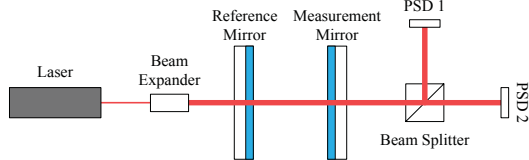
$$\mathcal{F}(\text{finesse}) = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}} = \frac{2\pi}{\Delta\nu_{1/2}} \quad (3)$$

## 3. INTERFEROMETRIC SIGNAL PROCESSING

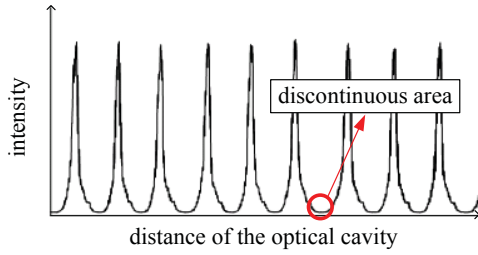
### 3.1. Interferometric system structure

The arranged interferometric system is shown in Fig. 4. The common sensing element of the interferometer is photodiode (PD). But for Fabry-Perot interferometer, the signal detected by PD will emerge a problem of the signals with discontinuous area (Fig. 5). The discontinuous area will be impossible for signal interpolation. For this reason, two position sensitive detectors (PSDs) are used as the sensors of this system. PSD is a kind of sensor which can reveal the center of position of the luminous intensity distribution on the sensing area. The fringe distance of the Fabry-Perot interferometer is adjusted to be equal to the width of sensing area of the PSD by regulation of the parallelism of the optical cavity. As shown in Fig. 6, two PSDs are arranged to detect interferometric signals with the phase shift of a quarter periods, such that two orthogonal signals with

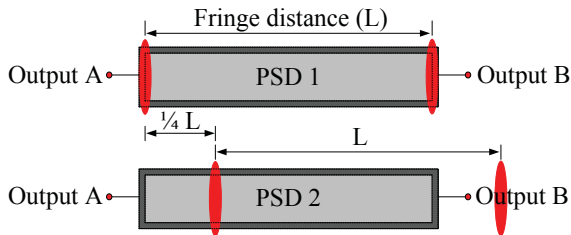
the phase difference of 90 degrees can be utilized for displacement measurement of the measurement mirror. And the signal detected by PSD is shown in Fig. 7. By this way, the continuous orthogonal signal can be obtained.



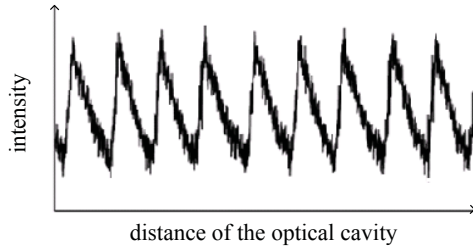
**Figure 4 - Interferometric system**



**Figure 5 – Signal detected by the PD**



**Figure 6 – Two PSDs and fringe distributions**



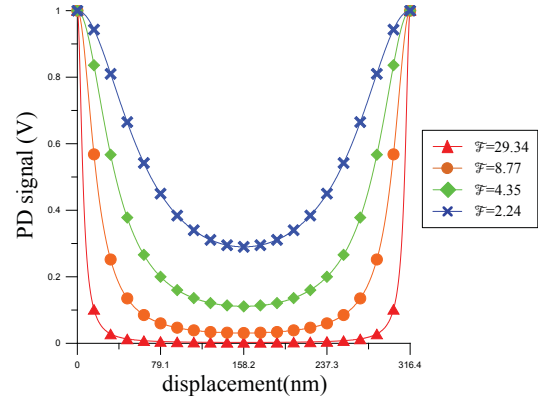
**Figure 7 – Signal detected by the PSD**

### 3.2. Interferometric signal model

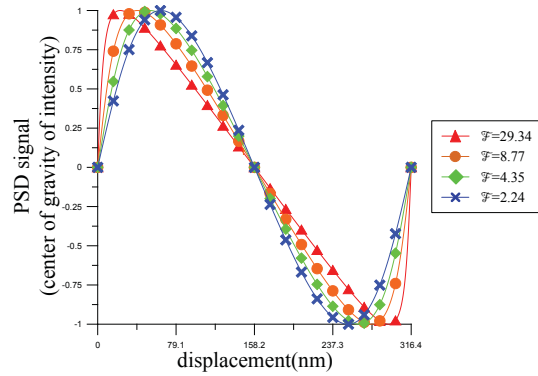
In Fabry-Perot interferometer, the interferometric signal will change with the different length of the optical cavity. So there must be an interpolation method which can vary with the different signal type. Here, the finesse is defined to be the only parameter of the interferometric signal. The signal interpolation model will change with the different finesse.

The signal which is detected by PD or PSD will different to each other. According to Fig. 8 and 9, the signals with different finesse are detected by PD and PSD. It is easy to observe that the variations of the signals with different finesse which are detected by PSD are less than that signal detected by PD. Fig. 10

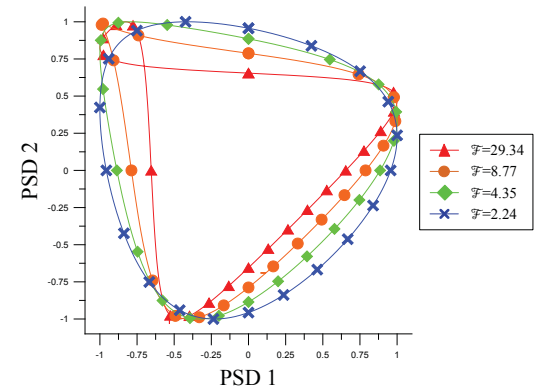
shows the Lissajou's figures of orthogonal signals which are detected by two PSDs.



**Figure 8 – Simulation signal for PD**



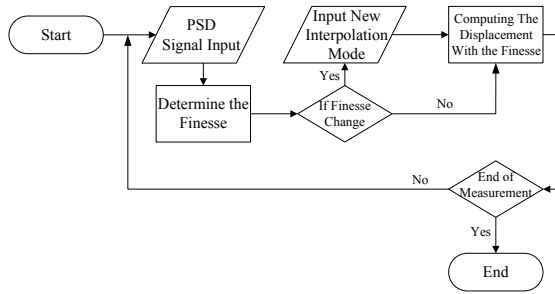
**Figure 9 – Simulation signal for PSD**



**Figure 10 – Lissajou's figure of simulation signals**

### 3.3. Signal interpolation method

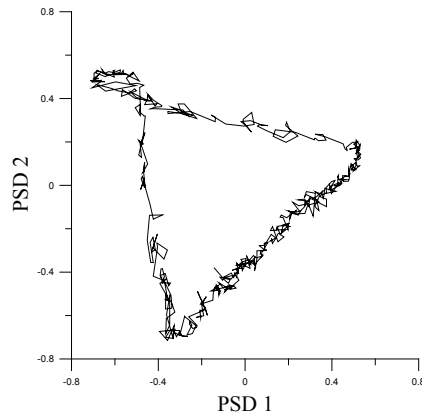
The flowchart of signal interpolation is shown in Fig. 11. At first, the signals which are detected by two PSDs are inputted into the signal processing system. Second, the system will determine the finesse of the signal. If the finesse is change, the system will import another interpolation module for the present finesse. At last, the system will compute the displacement with the present finesse. By this way, an algorithm for the signal interpolation of the Fabry-Perot interferometer can be realized.



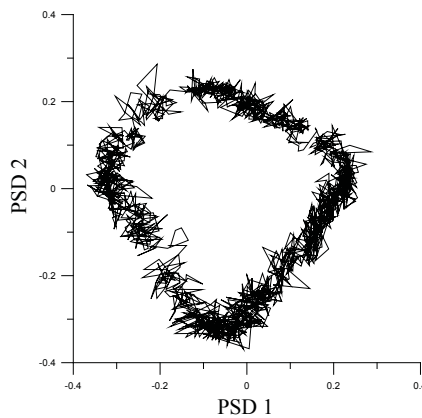
**Figure 11** – Flowchart of signal interpolation

#### 4. ANALYSIS AND RESULT

Fig. 12 and 13 are the Lissajou's figures of the experimental signals with different length of the optical cavity. The Fig. 12 is the Lissajou's figure with the higher finesse. It is similar to the simulation Lissajou's figures with the high finesse in the Fig. 10. And Fig. 13 is similar to the low finesse pattern. The result shows that the length of the optical cavity will influence the finesse sharply. And the finesse can represent the form of the Lissajou's figure.



**Figure 12** – Lissajou's figure of experimental signals with short distance of optical cavity (about 1 mm)



**Figure 13** – Lissajou's figure of experimental signals with large distance of optical cavity (about 50 mm)

#### 5. CONCLUSIONS

From the experimental results and analyses, some point can be summarized.

- Finesse is one of the most significant parameters of the Fabry-Perot interferometer. It can be used to represent the signal of the Fabry-Perot interferometer.
- By this signal interpolation method, the decadence of the signal is no longer a critical problem of the displacement measurement Fabry-Perot interferometer.
- Futurework, to verify the performance of the novel interpolation algorithm with the folded Fabry-Perot interferometer by the displacement experiment should be done.

#### 6. ACKNOWLEDGMENTS

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